

Supplementary material for “Spatially Localized Quasicrystals”

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In this supplementary material we present a video that explores the 3D structure of both the extended QC and a related spatially localized QC in more detail. The video presents data of two converged 3D equilibria of the model obtained through an iterative Newton method carried out in a periodic domain of size $(16 \times 2\pi)^3$. The system parameters are $r_1 = r_q = -0.51$, $q = 0.6180 = 1/\tau_3$, $Q = 2$, $\sigma_0 = -10$ and $\mu = 0$, just as in Fig. 1 of the main paper. In both states, $U(\mathbf{x})$ varies in the range $-0.36 \leq U(\mathbf{x}) \leq 2.26$ (shown in color on spheres of different radii), and we show isosurfaces of $U(\mathbf{x}) = 1.5$ as red blobs in all the figures. The video starts from the center of the domain, where there is a maximum of $U(\mathbf{x})$, with the 3D QC on the left and the spatially localized QC on the right. Four frames from the video are reproduced in Figures 1 and 2.

We note the different polyhedral arrangements that are formed by the isosurfaces as we move radially outward. Figure 1 shows the first two such structures, with icosahedra at radius $1.9 \times 2\pi$ (left panel) and rhombic triacontahedra at radius $3.7 \times 2\pi$ (right panel). At a radius of $5.1 \times 2\pi$ (Figure 2, left panel), the extended QC and spatially localized QCs are similar but the spatially localized QC has lower amplitude. At a radius of $6.7 \times 2\pi$ (right panel), the extended QC continues with icosahedral symmetry, while the spatially localized QC has very little variation in $U(\mathbf{x})$, and is surrounded by liquid.

The data for this paper, including a file that allows interaction with the data behind the extended and spatially localized QC states through the ParaView software, is available via the Research Data Leeds repository.

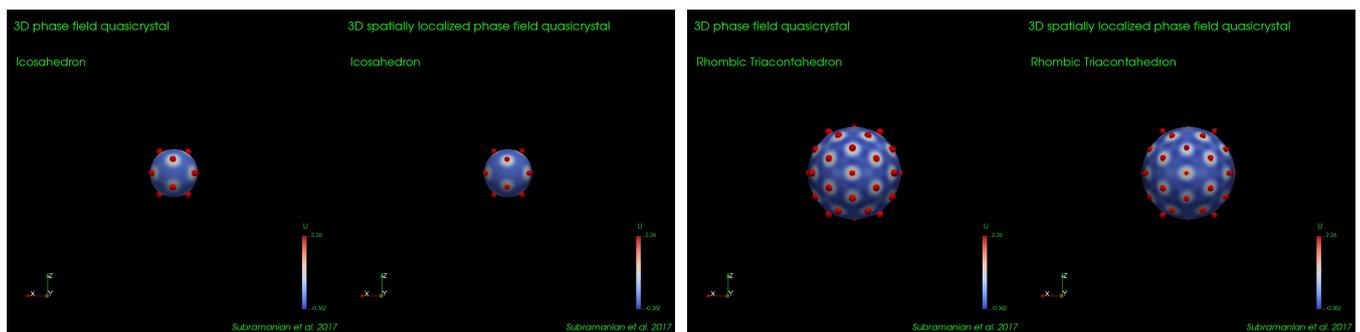


FIG. 1: Structure of both the 3D QC (left image in each panel) and the spatially localized QC (right image in each panel) depicted on spheres of two different sizes, centered on the middle of the periodic domain. The color bar indicates the variation of $U(\mathbf{x})$ on the spheres. The red blobs are isosurfaces of $U(\mathbf{x}) = 1.5$. Left panel: Both equilibria form icosahedra with 12 vertices (8 are observable in this view) at a radius of $1.9 \times 2\pi$. Right panel: on a sphere of radius $3.7 \times 2\pi$, $U(\mathbf{x})$ forms a rhombic triacontahedron with 32 vertices in both cases, with only slight differences.

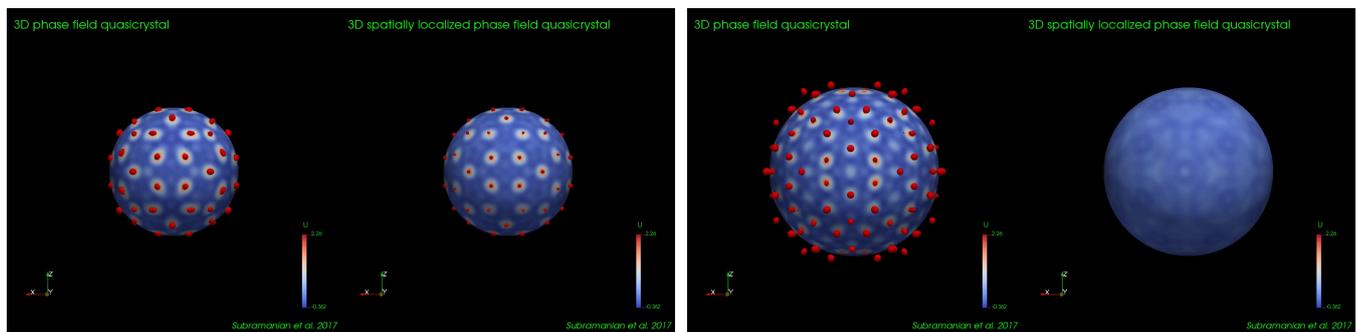


FIG. 2: Left panel: At a radius of $5.1 \times 2\pi$, both states show similar patterns but with different amplitudes, as reflected in the size of the red isosurfaces. Right panel: at radius $6.7 \times 2\pi$, the extended state continues with icosahedral symmetry, while the spatial structure in the spatially localized QC has vanished: this state is surrounded by liquid (blue) with only small variations in $U(\mathbf{x})$.